**Internal Rate of Return**

**Definition**

The internal rate of return on an investment or project is the "annualized effective compounded return rate" or "rate of return" that makes the [net present value](http://en.wikipedia.org/wiki/Net_present_value) (NPV as NET\*1/(1+IRR)^year) of all cash flows (both positive and negative) from a particular investment equal to zero.

In more specific terms, the IRR of an investment is the [discount rate](http://en.wikipedia.org/wiki/Discount_rate) at which the [net present value](http://en.wikipedia.org/wiki/Net_present_value) of costs (negative cash flows) of the investment equals the [net present value](http://en.wikipedia.org/wiki/Net_present_value) of the benefits (positive cash flows) of the investment.

IRR calculations are commonly used to evaluate the desirability of investments or projects. The higher a project's IRR, the more desirable it is to undertake the project. Assuming all projects require the same amount of up-front investment, the project with the highest IRR would be considered the best and undertaken first.

A firm (or individual) should, in theory, undertake all projects or investments available with IRRs that exceed the [cost of capital](http://en.wikipedia.org/wiki/Cost_of_capital). Investment may be limited by availability of funds to the firm and/or by the firm's capacity or ability to manage numerous projects.

**Uses**

Because the internal rate of return is a [rate](http://en.wikipedia.org/wiki/Rate_(mathematics)) quantity, it is an indicator of the efficiency, quality, or [yield](http://en.wikipedia.org/wiki/Yield_(finance)) of an investment. This is in contrast with the net present value, which is an indicator of the value or [magnitude](http://en.wikipedia.org/wiki/Magnitude_(mathematics)) of an investment.

An investment is considered acceptable if its internal rate of return is greater than an established [minimum acceptable rate of return](http://en.wikipedia.org/wiki/Minimum_acceptable_rate_of_return) or [cost of capital](http://en.wikipedia.org/wiki/Cost_of_capital). In a scenario where an investment is considered by a firm that has [equity holders](http://en.wikipedia.org/wiki/Equity_investment), this minimum rate is the [cost of capital](http://en.wikipedia.org/wiki/Cost_of_capital) of the investment (which may be determined by the risk-adjusted cost of capital of alternative investments). This ensures that the investment is supported by equity holders since, in general, an investment whose IRR exceeds its cost of capital adds [value](http://en.wikipedia.org/wiki/Value_(economics)) for the company (i.e., it is economically profitable).

The rate of return that equates the present value of a project’s cash inflows with the present value of its cash outflows i.e. it sets out the net present value equal to zero. Internal rate of return is basically used to measure the efficiency of capital investment. Internal rate of return is generally required low cost of capital to accept the project.

**Calculation**

Given a collection of pairs ([time](http://en.wikipedia.org/wiki/Time), [cash flow](http://en.wikipedia.org/wiki/Cash_flow)) involved in a project, the internal rate of return follows from the [net present value](http://en.wikipedia.org/wiki/Net_present_value) as a function of the [rate of return](http://en.wikipedia.org/wiki/Rate_of_return). A rate of return for which this function is zero is an internal rate of return.

Given the (period, cash flow) pairs (n, C_n) where nis a positive integer, the total number of periods N, and the net present value \mathrm{NPV}, the internal rate of return is given by rin:

\mathrm{NPV} = \sum_{n=0}^{N} \frac{C_n}{(1+r)^{n}}

The period is usually given in years, but the calculation may be made simpler if ris calculated using the period in which the majority of the problem is defined (e.g., using months if most of the cash flows occur at monthly intervals) and converted to a yearly period thereafter.

Any fixed time can be used in place of the present (e.g., the end of one interval of an [annuity](http://en.wikipedia.org/wiki/Annuity_(finance_theory))); the value obtained is zero if and only if the NPV is zero.

In the case that the cash flows are [random variables](http://en.wikipedia.org/wiki/Random_variable), such as in the case of a [life annuity](http://en.wikipedia.org/wiki/Life_annuity), the [expected values](http://en.wikipedia.org/wiki/Expected_value) are put into the above formula.

Often, the value of rcannot be found analytically. In this case, [numerical methods](http://en.wikipedia.org/wiki/Numerical_analysis) or [graphical methods](http://en.wikipedia.org/wiki/Plot_(graphics)) must be used.

**Example**

If an investment may be given by the sequence of cash flows

|  |  |
| --- | --- |
| Year (n) | Cash flow (C_n) |
| 0 | -1000000 |
| 1 | 300000 |
| 2 | 500000 |
| 3 | 500000 |

then the IRR ris given by

\mathrm{NPV} = -1000000+\frac{300000}{(1+r)^1} + \frac{500000}{(1+r)^2} + \frac{500000}{(1+r)^3} = 0.

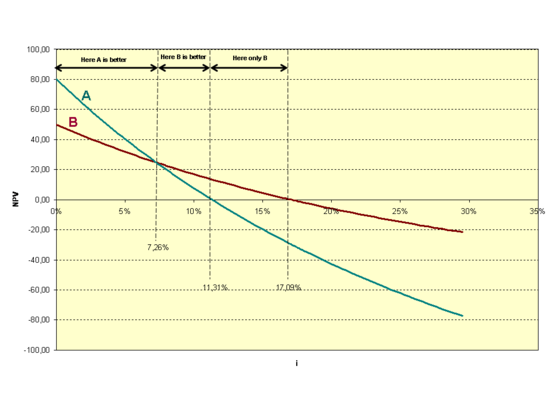
In this case, the answer is 13.19% (in the calculation, that is, r = .1319).

**Decision criterion**

* If the IRR is greater than the cost of capital, accept the project.
* If the IRR is less than the cost of capital, reject the project.

**Problems with using internal rate of return**

* As an [investment](http://en.wikipedia.org/wiki/Investment) decision tool, the calculated IRR should not be used to rate mutually exclusive projects, but only to decide whether a single project is worth investing in.

[](http://en.wikipedia.org/wiki/File:Exclusive_investments.png)

* NPV vs discount rate comparison for two mutually exclusive projects. Project 'A' has a higher NPV (for certain discount rates), even though its IRR (= x-axis intercept) is lower than for project 'B'.
* In cases where one project has a higher initial investment than a second mutually exclusive project, the first project may have a lower IRR (expected return), but a higher NPV (increase in shareholders' wealth) and should thus be accepted over the second project (assuming no capital constraints).
* IRR assumes reinvestment of interim cash flows in projects with equal rates of return (the reinvestment can be the same project or a different project). Therefore, IRR overstates the annual equivalent rate of return for a project whose interim cash flows are reinvested at a rate lower than the calculated IRR. This presents a problem, especially for high IRR projects, since there is frequently not another project available in the interim that can earn the same rate of return as the first project.
* When the calculated IRR is higher than the true reinvestment rate for interim cash flows, the measure will overestimate — sometimes very significantly — the annual equivalent return from the project. The formula assumes that the company has additional projects, with equally attractive prospects, in which to invest the interim cash flows.[[3]](http://en.wikipedia.org/wiki/Internal_rate_of_return#cite_note-cautionary-3)
* This makes IRR a suitable (and popular) choice for analyzing [venture capital](http://en.wikipedia.org/wiki/Venture_capital) and other [private equity](http://en.wikipedia.org/wiki/Private_equity) investments, as these strategies usually require several cash investments throughout the project, but only see one cash outflow at the end of the project (e.g., via [IPO](http://en.wikipedia.org/wiki/Initial_Public_Offering) or [M&A](http://en.wikipedia.org/wiki/M%26A)).
* Since IRR does not consider [cost of capital](http://en.wikipedia.org/wiki/Cost_of_capital), it should not be used to compare projects of different duration. [Modified Internal Rate of Return](http://en.wikipedia.org/wiki/Modified_Internal_Rate_of_Return) (MIRR) does consider cost of capital and provides a better indication of a project's efficiency in contributing to the firm's discounted cash flow.
* In the case of positive cash flows followed by negative ones and then by positive ones (for example, + + − − − +) the IRR may have multiple values. In this case a discount rate may be used for the borrowing cash flow and the IRR calculated for the investment cash flow. This applies for example when a customer makes a deposit before a specific machine is built.
* In a series of cash flows like (−10, 21, −11), one initially invests money, so a high rate of return is best, but then receives more than one possesses, so then one owes money, so now a low rate of return is best. In this case it is not even clear whether a high or a low IRR is better. There may even be multiple IRRs for a single project, like in the example 0% as well as 10%. Examples of this type of project are [strip mines](http://en.wikipedia.org/wiki/Strip_mine) and [nuclear power](http://en.wikipedia.org/wiki/Nuclear_power) plants, where there is usually a large cash outflow at the end of the project.

**NPV**

**Formula**

Each cash inflow/outflow is [discounted](http://en.wikipedia.org/wiki/Discounted) back to its present value (PV). Then they are summed. Therefore NPV is the sum of all terms,

\frac{R_t}{(1+i)^{t}}

Where

t- the time of the cash flow

* - the [discount rate](http://en.wikipedia.org/wiki/Discount_rate) (the [rate of return](http://en.wikipedia.org/wiki/Rate_of_return) that could be earned on an investment in the financial markets with similar risk.); the opportunity cost of capital

- the net cash flow i.e. cash inflow – cash outflow, at time t . For educational purposes, R_0 is commonly placed to the left of the sum to emphasize its role as (minus) the investment.



The result of this formula is multiplied with the Annual Net cash in-flows and reduced by Initial Cash outlay the present value but in cases where the cash flows are not equal in amount, then the previous formula will be used to determine the present value of each cash flow separately. Any cash flow within 12 months will not be discounted for NPV purpose, nevertheless the usual initial investments during the first year R0 are summed up a negative cash flow.

Given the (period, cash flow) pairs (t, R_t) where Nis the total number of periods, the net present value \mathrm{NPV}is given by:

\mathrm{NPV}(i, N) = \sum_{t=0}^{N} \frac{R_t}{(1+i)^{t}}

**The discount rate**

The rate used to discount future cash flows to the present value is a key variable of this process.

A firm's [weighted average cost of capital](http://en.wikipedia.org/wiki/Weighted_average_cost_of_capital) (after tax) is often used, but many people believe that it is appropriate to use higher discount rates to adjust for risk or other factors. A variable discount rate with higher rates applied to cash flows occurring further along the time span might be used to reflect the [yield curve](http://en.wikipedia.org/wiki/Yield_curve) premium for long-term debt.

Another approach to choosing the discount rate factor is to decide the rate which the capital needed for the project could return if invested in an alternative venture. If, for example, the capital required for Project A can earn 5% elsewhere, use this discount rate in the NPV calculation to allow a direct comparison to be made between Project A and the alternative. Related to this concept is to use the firm's reinvestment rate. Reinvestment rate can be defined as the rate of return for the firm's investments on average. When analyzing projects in a capital constrained environment, it may be appropriate to use the reinvestment rate rather than the firm's weighted average cost of capital as the discount factor. It reflects opportunity cost of investment, rather than the possibly lower cost of capital.

An NPV calculated using variable discount rates (if they are known for the duration of the investment) better reflects the situation than one calculated from a constant discount rate for the entire investment duration.

For some professional investors, their investment funds are committed to target a specified rate of return. In such cases, that rate of return should be selected as the discount rate for the NPV calculation. In this way, a direct comparison can be made between the profitability of the project and the desired rate of return.

To some extent, the selection of the discount rate is dependent on the use to which it will be put. If the intent is simply to determine whether a project will add value to the company, using the firm's weighted average cost of capital may be appropriate. If trying to decide between alternative investments in order to maximize the value of the firm, the corporate reinvestment rate would probably be a better choice.

Using variable rates over time, or discounting "guaranteed" cash flows differently from "at risk" cash flows, may be a superior methodology but is seldom used in practice. Using the discount rate to adjust for risk is often difficult to do in practice (especially internationally) and is difficult to do well. An alternative to using discount factor to adjust for risk is to explicitly correct the cash flows for the risk elements using [rNPV](http://en.wikipedia.org/wiki/RNPV) or a similar method, then discount at the firm's rate.

**Use in decision making**

NPV is an indicator of how much value an investment or project adds to the firm. With a particular project, if R_tis a positive value, the project is in the status of positive cash inflow in the time of t. If R_tis a negative value, the project is in the status of discounted cash outflow in the time of t.

Appropriately risked projects with a positive NPV could be accepted. This does not necessarily mean that they should be undertaken since NPV at the cost of capital may not account for [opportunity cost](http://en.wikipedia.org/wiki/Opportunity_cost), i.e., comparison with other available investments. In financial theory, if there is a choice between two mutually exclusive alternatives, the one yielding the higher NPV should be selected.

|  |  |  |
| --- | --- | --- |
| If... | It means... | Then... |
| NPV > 0 | the investment would add value to the firm | the project may be accepted |
| NPV < 0 | the investment would subtract value from the firm | the project should be rejected |
| NPV = 0 | the investment would neither gain nor lose value for the firm | We should be indifferent in the decision whether to accept or reject the project. This project adds no monetary value. Decision should be based on other criteria, e.g., strategic positioning or other factors not explicitly included in the calculation. |

The above strategy (pick the highest positive NPV) comes with some assumptions:

* Cash flow projection accuracy is expected to be the same across all possible projects (e.g. raw material and labor costs are expected to be about equally stable and able to be reliably forecast over the life of the various competing capital projects.)
* The investment horizon of all possible projects are equally acceptable (e.g. a 3 year project is not preferable vs. a 20 year project.)
* The 10% discount rate is the appropriate rate to discount the cash flows from each project. Each project is assumed equally speculative.
* the shareholders can't get above a 10% return on their money if they were to directly assume an equivalent level of risk. (If the investor could do better elsewhere, no projects should be undertaken by the firm and the money should be given back to the shareholder through dividends and stock repurchases.)
* More realistic problems would also need to consider other factors, generally including: smaller time buckets, the calculation of taxes (including the cash flow timing), inflation, currency exchange fluctuations, hedged or unhedged commodity costs, risks of technical obsolescence, potential future competitive factors, uneven or unpredictable [cash flows](http://en.wikipedia.org/wiki/Cash_flows), and a more realistic [salvage value](http://en.wikipedia.org/wiki/Salvage_value) assumption, as well as many others.

**Common pitfalls**

* If, for example, the R_tare generally negative late in the project (e.g., an industrial or mining project might have clean-up and restoration costs), then at that stage the company owes money, so a high discount rate is not cautious but too optimistic. Some people see this as a problem with NPV. A way to avoid this problem is to include explicit provision for financing any losses after the initial investment, that is, explicitly calculate the cost of financing such losses.
* Another common pitfall is to adjust for risk by adding a premium to the discount rate. Whilst a bank might charge a higher rate of interest for a risky project, that does not mean that this is a valid approach to adjusting a net present value for risk, although it can be a reasonable approximation in some specific cases. One reason such an approach may not work well can be seen from the following: if some risk is incurred resulting in some losses, then a discount rate in the NPV will reduce the impact of such losses below their true financial cost. A rigorous approach to risk requires identifying and valuing risks explicitly, e.g., by actuarial or [Monte Carlo](http://en.wikipedia.org/wiki/Monte_Carlo_method) techniques, and explicitly calculating the cost of financing any losses incurred.
* Yet another issue can result from the compounding of the risk premium. R is a composite of the risk free rate and the risk premium. As a result, future cash flows are discounted by both the [risk-free rate](http://en.wikipedia.org/wiki/Risk-free_rate) as well as the risk premium and this effect is compounded by each subsequent cash flow. This compounding results in a much lower NPV than might be otherwise calculated. The [certainty equivalent](http://en.wikipedia.org/wiki/Certainty_equivalent) model can be used to account for the risk premium without compounding its effect on present value.[[citation needed](http://en.wikipedia.org/wiki/Wikipedia:Citation_needed)]
* Another issue with relying on NPV is that it does not provide an overall picture of the gain or loss of executing a certain project. To see a percentage gain relative to the investments for the project, usually, [Internal rate of return](http://en.wikipedia.org/wiki/Internal_rate_of_return) or other efficiency measures are used as a complement to NPV.
* Non-specialist users frequently make the error of computing NPV based on cash flows after interest. This is wrong because it double counts the time value of money. Free cash flow should be used as the basis for NPV computations.

**MIRR**

**Background**

Net present value as a valuation methodology dates at least to the 19th century. [Karl Marx](http://en.wikipedia.org/wiki/Karl_Marx) refers to NPV as [fictitious capital](http://en.wikipedia.org/wiki/Fictitious_capital), and the calculation as "capitalising," writing:

“The forming of a fictitious capital is called capitalising. Every periodically repeated income is capitalised by calculating it on the average rate of interest, as an income which would be realised by a capital at this rate of interest.”

In [mainstream](http://en.wikipedia.org/wiki/Mainstream_economics) [neo-classical economics](http://en.wikipedia.org/wiki/Neo-classical_economics), NPV was formalized and popularized by [Irving Fisher](http://en.wikipedia.org/wiki/Irving_Fisher), in his 1907 The Rate of Interest and became included in textbooks from the 1950s onwards, starting in finance texts.

**MIRR**

**Problems with the IRR**

While there are several [problems with the IRR](http://en.wikipedia.org/wiki/Internal_rate_of_return#Problems_with_using_internal_rate_of_return), MIRR resolves two of them.

* Firstly, IRR assumes that interim positive cash flows are reinvested at the same rate of return as that of the project that generated them.[[3]](http://en.wikipedia.org/wiki/Modified_Internal_Rate_of_Return#cite_note-3) This is usually an unrealistic scenario and a more likely situation is that the funds will be reinvested at a rate closer to the firm's cost of capital. The IRR therefore often gives an unduly optimistic picture of the projects under study. Generally for comparing projects more fairly, the [weighted average cost of capital](http://en.wikipedia.org/wiki/Weighted_average_cost_of_capital) should be used for reinvesting the interim cash flows.
* Secondly, more than one IRR can be found for projects with alternating positive and negative cash flows, which leads to confusion and ambiguity. MIRR finds only one value.

**Calculation of the MIRR**

MIRR is calculated as follows:

* where n is the number of equal periods at the end of which the cash flows occur (not the number of cash flows), PV is [present value](http://en.wikipedia.org/wiki/Present_value) (at the beginning of the first period), FV is [future value](http://en.wikipedia.org/wiki/Future_value) (at the end of the last period).
* The formula adds up the negative cash flows after discounting them to time zero using the external cost of capital, adds up the positive cash flows including the proceeds of reinvestment at the external reinvestment rate to the final period, and then works out what rate of return would cause the magnitude of the discounted negative cash flows at time zero to be equivalent to the future value of the positive cash flows at the final time period.

[Spreadsheet applications](http://en.wikipedia.org/wiki/Spreadsheet), such as [Microsoft Excel](http://en.wikipedia.org/wiki/Microsoft_Excel), have inbuilt functions to calculate the MIRR. In Microsoft Excel this function is "=MIRR".

Example

If an investment project is described by the sequence of cash flows:

|  |  |
| --- | --- |
| Year | Cash flow |
| 0 | -1000 |
| 1 | -4000 |
| 2 | 5000 |
| 3 | 2000 |

then the IRR ris given by

In this case, the answer is 25.48% (the other solutions to this equation are -593.16% and -132.32%, but they will not be considered meaningful IRRs).

To calculate the MIRR, we will assume a finance rate of 10% and a reinvestment rate of 12%. First, we calculate the present value of the negative cash flows (discounted at the finance rate):

Second, we calculate the future value of the positive cash flows (reinvested at the reinvestment rate):

Third, we find the MIRR:

The calculated MIRR (17.91%) is significantly different from the IRR (25.48%).

**Comparing projects of different sizes**

Like the internal rate of return, the modified internal rate of return cannot be validly used to rank-order projects of different sizes, because a larger project with a smaller modified internal rate of return may have a higher present value. However, there exist variants of the modified internal rate of return which can be used for such comparisons.[[](http://en.wikipedia.org/wiki/Modified_Internal_Rate_of_Return#cite_note-4)